32.4). Note that a sharply defined heating wavefront (with infinite derivatives) exists only, if condition $\alpha + \beta > 1$ is satisfied. For $\alpha + \beta \leqslant 1$ the front is not sharply defined.

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ON EXTERNAL FLOWS INDUCED BY JETS IN A VISCOUS INCOMPRESSIBLE FLUID

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A method is proposed for calculating flows induced by fluid sucked into a jet outside the boundary layer region. The jet is simulated by a set of sinks whose intensity is specified in terms of known solutions of the boundary layer theory.

With few exceptions [1 – 3] problems of the boundary layer theory related to jet-like flows of viscous fluids are solved for high Reynolds numbers. In such approximation the presence of suction of fluid into the jet is a distinctive feature, which has the effect of inducing motion of the fluid in the space outside the jet. Since in the external region of flow the velocities of fluid motion and the Reynolds numbers are not high, the inertia terms in the Navier-Stokes equations can be neglected. A fine jet can be simulated by a system of sinks distributed along its axis. The intensity of sinks is deremined by solutions which define jet-like flows within the limits of the boundary layer theory [4, 5]. The linear problem of external flow thus formulated can be analytically solved for various kinds of jet-like fluid motions. Several examples are presented.

1. External flow induced by a jet flowing from a narrow tube. We introduce a system of spherical coordinates with origin at the jet outlet and angle θ measured from the jet axis. We seek components of velocity and pressure in the form

$$v_r = \frac{\nu}{r} f(\theta), \quad v_\theta = \frac{\nu}{r} \varphi(\theta), \quad \frac{P}{\rho} = \frac{\nu^2}{r^2} F(\theta) \tag{1.1}$$

Linearizing the system of equations of motion [6], we have

$$f'' + f'\mathrm{ctg}\theta + 2F = 0$$

$$-f' + F' = 0, \quad f + \varphi' + \varphi\,\mathrm{ctg}\;\theta = 0 \tag{1.2}$$

Eliminating f and F, we obtain the equation

$$\varphi'\sin^2\theta - \varphi\sin\theta\cos\theta = M\cos 2\theta + N\cos\theta + R \tag{1.3}$$

where M, N and R are arbitrary constants of integration. The general solution of (1.3) can be written as

$$\varphi = A_1\sin\theta + \frac{A_2}{\sin\theta} + A_3\,\mathrm{ctg}\,\theta + A_4\sin\theta\ln\left(\mathrm{ctg}\,\frac{\theta}{2}\right) \tag{1.4}$$

where A_1, A_2, A_3 and A_4 are certain new constants. From the last equation of system (1.2) we obtain

$$f = -2A_1\cos\theta + A_3 - A_4\left[2\cos\theta\ln\left(\mathrm{ctg}\,\frac{\theta}{2}\right) - 1\right]$$

Let us consider a jet propagating in a boundless space [1]. From the condition that v_θ vanishes for $\theta = \pi$ and that v_r is bounded for $\theta = 0$ we have $A_2 = A_3$ and $A_4 = 0$. Hence

$$\varphi = A_1\sin\theta + A_3\,\mathrm{ctg}\,\frac{\theta}{2} \tag{1.5}$$

We determine constant A_3 by the condition that the total discharge of fluid through the spherical surface S of arbitrary radius must be zero. Taking into consideration that the quantity $8\pi\nu r$ of fluid [4] is discharged through the part of the sphere S_0 equal to the jet cross section, and neglecting the area of S_0 which is small in comparison with S, we obtain

$$\frac{\partial}{\partial r}\int_0^\pi 2\pi r^2 v_r\sin\theta\,d\theta = -8\pi\nu$$

Using the equation of continuity, we reduce this relationship to

$$\lim_{\theta\to 0} 2\pi r\sin\theta\, v_\theta = -8\pi\nu \tag{1.6}$$

Substituting $v_\theta = (\nu / r)\,\varphi(\theta)$ into (1.6) and allowing for (1.5), we obtain $A_3 = -2$. The stream of external flow momentum is defined by

$$I_1 = 2\pi r^2\int_0^\pi \Pi_{rr}\cos\theta\sin\theta\,d\theta = 2\pi r^2\int_0^\pi\left(-P + 2\eta\frac{\partial v_r}{\partial r}\right)\cos\theta\sin\theta\,d\theta$$

Setting $I_1 = 0$, we obtain $A_1 = 0$. Thus the velocity components of the external flow are defined by

$$v_r = -\frac{2\nu}{r}, \qquad v_\theta = -\frac{2\nu}{r}\,\mathrm{ctg}\,\frac{\theta}{2} \tag{1.7}$$

In the case of a strong jet and angles θ such that $\sin^{1}/_{2}\theta \gg \alpha / 2$, where $\alpha = 32\nu^2\rho / (3I_0)$ and I_0 is the jet momentum, formula (1.7) is exactly the same as the solution derived in [1].

Solution (1.7) for the radial velocity component (curve *1*) is shown in Fig. 1. Curves *2* and *3* in that figure represent, respectively, the exact solution of the complete Navier-Stokes equations and the solution of equations of the boundary layer [4]. Numerical calculations were carried out for $\alpha = 0.25$ and $U_r = 2\nu/r$.

2. External flow induced by a jet flowing along the axis of a conical diffuser with a 2β vertex angle. We determine the unknown constants A_1, A_2, A_3 and A_4 of solution (1.4), using the following boundary conditions: velocity components must vanish for $\theta = \beta$ and v_r must be bounded for $\theta = 0$. We have

$$A_3 = 2A_1 \cos\beta, \qquad A_2 = -A_1(1 + \cos^2\beta), \qquad A_4 = 0 \tag{2.1}$$

Substituting (2.1) into (1.4), we obtain

$$\varphi = -\frac{A_1}{\sin\theta}(\cos\theta - \cos\beta)^2$$

With the use of (1.6) we determine $A_1 = \sin^{-4} {}^1/_2\beta$. As the result, the velocity components take the form

$$v_r = -\frac{\nu}{r}\frac{2}{\sin^4 {}^1/_2\beta}(\cos\theta - \cos\beta)$$

$$v_\theta = -\frac{\nu}{r}\frac{1}{\sin^4 {}^1/_2\beta \sin\theta}(\cos\theta - \cos\beta)^2 \tag{2.2}$$

In the considered case $I_1 \neq 0$, and solution (2.2) is valid, when $I_1 / I_0 \ll 1$. From this we can obtain the condition $\alpha \ll {}^2/_3(1 - \cos\beta)$ which defines the limit of the diffuser cone-angle.

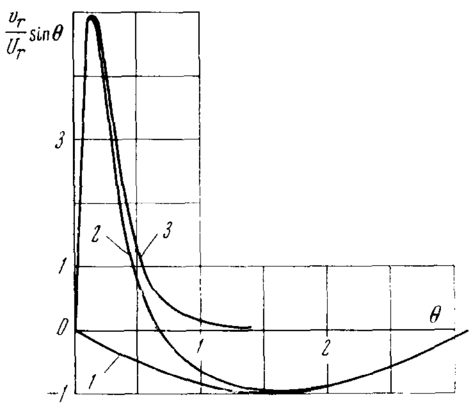

Fig. 1

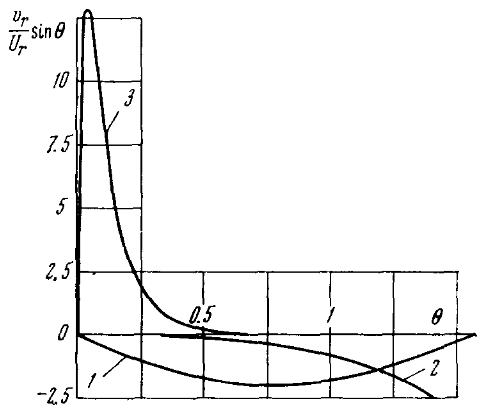

Fig. 2

Curve *1* in Fig. 2 represents solution (2.2) for the external flow induced by the jet for $\beta = \pi/2$, while curve *2* represents the solution obtained in [2] for $I_0 \to \infty$. Curve *3* was obtained by solving equations of the boundary layer [4] for $\alpha = 0.1$.

It should be pointed out that the difference between curves *1* and *2* is explained by that the solutions derived in [2, 3] do not satisfy the no-slip condition at the diffuser wall.

3. External flow induced by a jet flowing from a plane slot. We introduce a system of polar coordinates $r\varphi$ with origin at the jet outlet and angle φ measured from the jet axis. The linearized equation of motion for the stream function ψ is of the form

$$\Delta^2\psi = 0; \qquad v_r = \frac{1}{r}\frac{\partial\psi}{\partial\varphi}, \qquad v_\varphi = -\frac{\partial\psi}{\partial r} \tag{3.1}$$

Taking into account the solution obtained in [5], we seek the stream function ψ in the form

$$\psi = 1.65 \sqrt[3]{\frac{\nu I_0}{\rho}}\, r F(\varphi) \tag{3.2}$$

The substitution of (3.2) into (3.1) yields for F

$$F^{IV} + {}^{26}/_{9}F'' + {}^{25}/_{81}F = 0$$

The general solution of this equation is of the form

$$F = B_1 \sin {}^5/_3\varphi + B_2 \cos {}^5/_3\varphi + B_3 \sin {}^1/_3\varphi + B_4 \cos {}^1/_3\varphi$$

where B_1, B_2, B_3 and B_4 are constants of integration.

Let us first consider the case in which the jet propagates in an unbounded space. We specify the following boundary conditions:

$$v_r = 0, \qquad v_\varphi = -1.65 \sqrt[3]{\frac{\nu I_0}{\rho}\, r}, \quad \varphi = 0$$

$$v_\varphi = \frac{\partial v_r}{\partial \varphi} = 0, \qquad \varphi = \pi$$

Assuming that the intensity of fluid suction into the jet is known [5], for B_1, B_2, B_3 and B_4 we obtain the system of equations

$$\begin{gathered} 5B_1 + B_3 = 0, \quad B_2 + B_4 = 1 \\ -\sqrt{3}B_1 + B_2 + \sqrt{3}B_3 + B_4 = 0 \\ -25\sqrt{3}B_1 + 25B_2 + \sqrt{3}B_3 + B_4 = 0 \end{gathered} \tag{3.3}$$

Having determined B_1, B_2, B_3 and B_4 with the use of system (3.3), we calculate ψ by formula

$$\psi = \frac{1.65}{6} \sqrt[3]{\frac{\nu I_0}{\rho}}\, r \left[\frac{1}{\sqrt{3}} (\sin {}^5/_3\varphi + 5 \sin {}^1/_3\varphi) + (5 \cos {}^1/_3\varphi + \cos {}^5/_3\varphi)\right]$$

$$0 \leqslant \varphi \leqslant 2\pi$$

If the jet propagates in a diffuser with a 2γ vertex angle, then, by virtue of the condition that velocity components must vanish at the diffuser wall, we have

$$\begin{gathered} B_1 \sin {}^5/_3\gamma + B_2 \cos {}^5/_3\gamma + B_3 \sin {}^1/_3\gamma + B_4 \cos {}^1/_3\gamma = 0 \\ 5B_1 \cos {}^5/_3\gamma - 5B_2 \sin {}^5/_3\gamma + B_3 \cos {}^1/_3\gamma - B_4 \sin {}^1/_3\gamma = 0 \end{gathered} \tag{3.4}$$

Conditions for $\varphi = 0$ remain unchanged.

In the case of a jet flowing from a plane wall $(\gamma = \pi / 2)$ from (3.4) with allowance for (3.3) we have

$$\psi = \frac{1.65}{22} \sqrt[3]{\frac{\nu I_0}{\rho}}\, r\, [3\sqrt{3} (5 \sin {}^1/_3\varphi - \sin {}^5/_3\varphi) + (17 \cos {}^5/_3\varphi + 5 \cos {}^1/_3\varphi)]$$

These examples show that the proposed method can be used for the determination of external flows induced by jets.

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THE BEHAVIOR OF A WING PANEL IN A STREAM OF GAS UNDER TRANSIENT CONDITIONS

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A method is proposed for investigating the behavior of an elastic flat panel of aircraft wing skin in a stream of gas under transient conditions. The panel is assumed to have an initial deflection. The solution is based on the wave equation of linearized unsteady aerodynamics and the geometrically nonlinear equations of the theory of elastic plates [1]. The resolving equations which define the behavior of an elastic system are derived with the use of the Bubnov-Galerkin procedure with respect to one of the coordinates and of the method of finite differences with respect to the other coordinate and time. As an example, the supersonic flow past a wing is considered. Aerodynamic pressure distribution over the panel surface is determined, using a thin lift surface as the model, by the numerical method of retarded source potential, taking into consideration previous history of the deformation process [2].

Let a wing of rectangular plan form, moving in the direction of its axis of symmetry at velocity U_0 and zero angle of attack in a perfect compressible fluid, be subjected at instant $t = 0$ to small additional transient motions caused by instantaneous variation of the angle of attack α induced by vertical gusts. It is assumed that in the following instants individual sections of the wing skin begin to distort, and that the flow around the wing is streamlined.

Let us analyze the dynamic reaction of an elastic system subjected to a sudden change of stream parameters on the example of a flat wing skin panel section of sides a and b and thickness h (Fig. 1). Let us assume that this plate is attached by hinges to the structure reinforcing members, is loaded in its plane by compressive stresses p, has an initial deflection, and lies in the downstream Mach cone. Note that in this case the end effects and the vortex sheet do not affect pressure distribution over the panel surface. The